NUMBER THEORY.

When this issue was made up no solutions had been received for numbers 189, 191, 192, 196, 201, and 202. Please give attention to these.

208. Proposed by E. T. BELL, Seattle, Washington.

If an odd number is perfect, it cannot be the sum of two squares.

209. Proposed by R. D. CARMICHAEL, Indiana University.

Prove that the difference of the sixth powers of two integers cannot be the square of an integer

210. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

If a and b are relatively prime and $(a+b)$ is even, then $(a-b)ab(a+b) \equiv 0 \pmod{24}$.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

394. Proposed by E. B. ESCOTT, Ann Arbor, Michigan.

Solve the equation $\sin^2 x \sin^2 2x = 5/16$.

SOLUTION BY R. M. MATHEWS, Riverside, California.

Taking the square root of each side of the given equation we obtain

$$(1)\ \sin x \sin 2x = \frac{\sqrt{5}}{4} \quad \text{and} \quad (2)\ \sin x \sin 2x = -\frac{\sqrt{5}}{4}.$$

From (1),

$$2\sin^2 x \cos x = \frac{\sqrt{5}}{4}, \quad \text{or} \quad (3)\ \cos^3 x - \cos x + \frac{\sqrt{5}}{8} = 0.$$

Inspection shows that $-\frac{\sqrt{5}}{2}$ is a value of $\cos x$ in (3). Eliminating this impossible solution, we have the quadratic

$$(4) \qquad \cos^2 x - \frac{\sqrt{5}}{2}\cos x + \frac{1}{4} = 0,$$

the roots of which are $\cos x = \frac{\sqrt{5}+1}{4}$ and $\cos x = \frac{\sqrt{5}-1}{4}$.

The solutions of (2) are the negatives of those of (1) just found.

The side of a regular inscribed decagon is $(\sqrt{5}-1)/2$, whence $(\sqrt{5}-1)/4 = \cos 72°$. From this, it is easily shown that $(\sqrt{5}+1)/4 = \cos 36°$.

Since in the original equations the functions are all squared, the signs which they may have in particular quadrants are immaterial. Hence

$$x = n\pi \pm \tfrac{1}{5}\pi, \quad x = n\pi \pm \tfrac{2}{5}\pi,$$

where n is any positive or negative integer, are the real values satisfying the equation.

Also solved by C. E. GITHENS, HORACE OLSON, ALBERT R. NAUER, RICHARD MORRIS, A. M. HARDING, ELMER SCHUYLER, S. W. REAVES, W. C. EELLS, LEROY M. COFFIN, A. H. HOLMES, F. M. MORGAN, H. C. FEEMSTER, J. L. RILEY, H. E. TREFETHEN, and the PROPOSER.

395. Proposed by V. M. SPUNAR, Chicago, Illinois.

Solve the system of equations

$$x_1^2 x_2 = a_1, \quad x_2^2 x_3 = a_2, \quad x_3^2 x_4 = a_3, \quad \cdots, \quad x_n^2 x_1 = a_n.$$